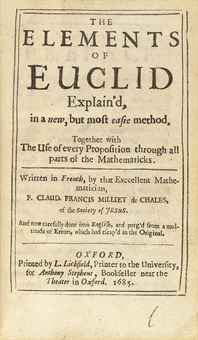
**Reasoning and Proof** Name:

For thousands of years, Babylonian, Egyptian, Chinese, and other mathematicians discovered geometry principles and developed ways to do practical geometry.

[](http://www.christies.com/lotfinder/books-manuscripts/euclid-milliet-dechales-claude-francois-milliet-5541389-details.aspx)

However, in 600 B.C.E., as civilization grew and ideas were exchanged and freely debated, people began insisting on reasons to support statements made in a debate. This is when mathematicians began to use logical reasoning to deduce mathematical ideas.

Greek mathematician Thales of Miletus made his geometry ideas convincing by supporting his discoveries with logical reasoning. Over the next 300 years, the process of supporting mathematical conjectures with logical arguments became more and more refined.

Pythagoras, Plato, and Euclid followed in Thales’ footsteps. Euclid, however, established a chain of arguments for the geometry that we know today.

How did Euclid do this?

Start with **premises** --> \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Premises organize geometry properties.

Then, form **postulates/axioms** --> \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Use postulates to form **theorems** -->

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So far in this class, you’ve been discovering geometry properties inductively, the way mathematicians have over centuries! (Ex: when you drew a bunch of lines and investigated what happened with angles and when you investigated slopes of parallel and perpendicular lines.)

Once you investigated something, you made a *conjecture* about what happened.

**\*** A **conjecture** is:

You then used an informal argument to explain why it might be true. However, it is important to prove your conjectures—show that they are indeed true.

How will you do that???

General process:

**Start with premises (definitions, properties, postulates)**

**Use premises to prove conjectures**

**Proved conjectures become theorems**

**Use theorems to prove other conjectures, and turn those into theorems**

In geometry, there are important premises to follow in order to build logical arguments:

* Definitions (they should be precise) and undefined terms (point, line, plane)
* Properties of arithmetic, equality, and congruence
* Postulates
* Previously proved conjectures (theorems)

**Properties of Arithmetic** for any numbers a, b, and c

Commutative Property: a + b = b + a ab = ba

Associative Property: (a + b) + c = a + (b + c) (ab)c = a(bc)

Distributive Property: a(b + c) = ab + bc

**Properties of Equality**

Reflective Property: a = a (anything is equal to itself)

Transitive Property: If a = b and b = c, then a = c.

Symmetric Property: If a = b, then b = a.

Addition Property: If a = b, then a + c = b + c.

Subtraction Property: If a = b, then a – c = b – c.

Multiplication Property: If a = b, then ac = bc.

Division Property: If a = b, then a/c = b/c.

Why are the above properties so important in geometry? Lengths of segments and measures of angles involve numbers, so you will need to use arithmetic and equality properties when proving many conjectures.

**Definition of Congruence**

If AB = CD, then AB CD. (Conversely: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)

If m∠A = m∠B, then ∠A ∠B. (Conversely: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_)

**Postulates of Geometry**

Line Postulate: You can construct exactly one line through any two points.

Line Intersection Postulate: The intersection of two distinct lines is exactly one point.

Duplication Postulate: You can construct a segment or angle congruent to another segment or angle.

Angle Bisector Postulate: You can construct exactly one angle bisector in any angle.

Parallel Postulate: Through a point not on a given line, you can construct exactly one line parallel to the given line.

Perpendicular Postulate: Through a point not on a given line, you can construct exactly one line perpendicular to the given line.

Segment Addition Postulate: If point B is on AC and between points A and C, then

AB + BC = AC.

Angle Addition Postulate:

m∠ABD + m∠DBC = m∠ABC

Linear Pair Postulate:

If two angles are a linear pair, then they are supplementary

Corresponding Angles Postulate:

If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

Conversely, if two lines are cut by a transversal forming congruent corresponding angles, then the lines are parallel.

**All About Assumptions**

When working with figures, it must be explicitly clear what we can and cannot assume from a diagram. Consider the following diagram to help us arrive at assumptions we can and cannot make:



**What we can assume: What we can’t assume:**

**Notation to remember**:

**Walking through a Proof**

When we prove the validity of a mathematical statement, we are then able to use this statement on future exercises where the statement could be of use. If the mathematical statement is likely to be used frequently as a building block tool for future endeavors, we call it a theorem. Before using a theorem, of course, we must prove its validity.

Let’s take a look at two simple proofs, knowing that we need to start at the most basic level, given that we have no theorems at our disposal to use.

The statement “If two angles are supplementary to congruent angles, then they are congruent” seems true, but we can’t just accept it to be true. Here’s how we’d prove it:



What do you notice about how this is written? (Note: You don’t always have to use this format.)

Let’s try this proof:



Let’s try this one, too:

